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Engineering Note

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Dynamic Thrust Offset Effects and Flying Qualities Implications

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Nomenclature

$\boldsymbol{A}(s)$ = coefficient matrix of homogeneous system
$\boldsymbol{B}$ = scaling matrix of control inputs
C_D = drag coefficient
C_{D_α} = $\partial C_D/\partial\alpha$
C_{D_δ} = $\partial C_D/\partial\delta_e$
C_L = lift coefficient
C_{L_α} = $\partial C_L/\partial\alpha$
C_{L_δ} = $\partial C_L/\partial\delta_e$
C_m = pitching moment coefficient
C_{m_q} = $\partial C_m/\partial(q\bar{c}/V_0)$
C_{m_V} = $\partial C_m/\partial(V/V_0)$
C_{m_α} = $\partial C_m/\partial\alpha$
$C_{m_{\dot\alpha}}$ = $\partial C_m/\partial(\dot\alpha\bar{c}/V_0)$
C_{m_δ} = $\partial C_m/\partial\delta_e$
$\bar{c}$ = mean aerodynamic chord
g = acceleration due to gravity
i_y = radius of gyration
M = pitching moment
n_V = thrust/speed dependence, $(V_0/T_0)\partial T/\partial V$
q = pitch rate
S = reference area
s = Laplace operator
T = thrust, time constant (with subscript)
$\boldsymbol{u}(s)$ = control vector
V = speed
$\boldsymbol{x}(s)$ = state vector
z_T = distance between thrust line of action and c.g.
α = angle of attack
γ = flight-path angle
Δ = perturbation, e.g., ΔV
δ_e = pitch control
δ_T = throttle position
μ = relative mass parameter, $2m/(\rho S\bar{c})$
ρ = air density
σ_p = phugoid damping
τ = reference time, $\mu\bar{c}/V_0$
ω_p = undamped natural phugoid frequency

Introduction

IN a recent paper,[1] the effects of thrust offset are considered. The influence of thrust on pitching moment is addressed, and the resulting changes in damping and stability are shown, with emphasis placed on the phugoid and static stabiliy for constant thrust jet aircraft. There are further papers[2–4] considering thrust offset effects, which are an issue of continuous interest.

The purpose of this Note is to show under which conditions thrust offset introduces dynamic instability, with particular emphasis placed on aperiodic instability. Further, the impact of thrust offset effects on flying qualities is considered. It is shown that there is an inconsistency of existing flying qualities criteria and requirements with regard to dynamics characteristics caused by thrust offset. In addition, degradations concerning flying qualities metrics are considered.

Modeling of Thrust Offset and Longitudinal Dynamics

Thrust offset, which denotes the thrust line of action relative to the c.g., basically introduces a pitching moment

$$M_T = z_T T \tag{1}$$

that is balanced by an aerodynamic moment in steady-state flight. When disturbed from trim, thrust offset causes a pitching moment change that may be described with the use of a stability derivative[2]

$$C_{m_V} = (n_V - 2)(z_T/\bar{c})C_D \tag{2}$$

For a general treatment of thrust/speed characteristics, $n_V = (V_0/T_0)\partial T/\partial V$ is introduced to describe the dependence of thrust on speed (or Mach number, respectively) to account for different propulsion systems such as constant thrust jet aircraft ($n_V \approx 0$) or propeller-driven vehicles ($n_V \approx -1$). There may be further thrust effects on pitching moment such as an influence on the flowfield, yielding an additional contribution to the pitching moment change. They are considered to be included in C_{m_V}.

With the use of the pitching moment derivative Eq. (2), the linearized equations of the longitudinal motion for a horizontal reference flight condition may be expressed as

$$\boldsymbol{A}(s)\dot{\boldsymbol{x}}(s) = \boldsymbol{B}\boldsymbol{u}(s) \tag{3}$$

where

$$\boldsymbol{x}(s) = [\Delta V/V_0, \Delta\alpha, \Delta\gamma]^T, \qquad \boldsymbol{u}(s) = [\delta_e, \delta_T]^T$$

$$\boldsymbol{A}(s) = \begin{bmatrix} s\tau + (2-n_V)C_D & C_{D_\alpha} & C_L \\ 2C_L & C_{L_\alpha} + C_D & -s\tau \\ \mu C_{m_V} & -(s\tau i_y/\bar{c})^2 + s\tau\left(C_{m_q} + C_{m_{\dot\alpha}}\right) + \mu C_{m_\alpha} & -(s\tau i_y/\bar{c})^2 + s\tau C_{m_q} \end{bmatrix}$$

$$\boldsymbol{B} = -\begin{bmatrix} C_{D_\delta} & -C_D \\ C_{L_\delta} & 0 \\ \mu C_{m_\delta} & \mu(z_T/\bar{c})C_D \end{bmatrix}$$

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Thrust Offset Effects on Longitudinal Dynamics

An approximate factorization of the characteristic equation shows that the primary effect of thrust offset is related to the phugoid, whereas the short period shows no or only little change. The following analytical expressions hold for phugoid frequency and damping:

$$\omega_p \approx (g/V_0)\sqrt{2 - \left(C_{L_\alpha}/C_L\right)\left(C_{m_V}/C_{m_\alpha}\right)}$$

$$2\sigma_p \approx (g/V_0)\left[-(2 - n_V)C_D + C_{D_\alpha}\left(C_{m_V}/C_{m_\alpha}\right)\right] \tag{4}$$

A primary aspect relates to the front- or the backside of the power-required curve because this is a decisive factor for amplifying or weakening the thrust offset effects. Operation on the backside of the power-required curve

$$C_{D_\alpha}/C_{L_\alpha} > (1 - n_V/2)C_D/C_L \tag{5}$$

yields an amplification of the destabilizing thrust offset effects ($C_{m_V} < 0$). By contrast, the frontside operation shows a weakened effect. The effects of thrust offset on aircraft dynamics as influenced by backside or frontside operation are shown in Figs. 1 and 2.

For backside operation, which is considered in the following, instability is introduced when C_{m_V} is more negative than the stability boundary value

$$\left(C_{m_V}\right)_{\sigma_p = 0} = (2 - n_V)\left(C_D/C_{D_\alpha}\right)C_{m_\alpha} \tag{6}$$

At the stability boundary, an undamped oscillation exists the frequency of which can be approximated by

$$\omega_p \approx (g/V_0)\sqrt{2 - (2 - n_V)(C_D/C_L)\left(C_{L_\alpha}/C_{D_\alpha}\right)} \tag{7}$$

The addressed effects are also illustrated in Fig. 1, which shows that negative C_{m_V} values beyond $(C_{m_V})_{\sigma_p=0}$ [Eq. (6)] introduce dynamic instability. Figure 1 further reveals that the instability mode first introduced is an oscillatory motion. This type of instability is

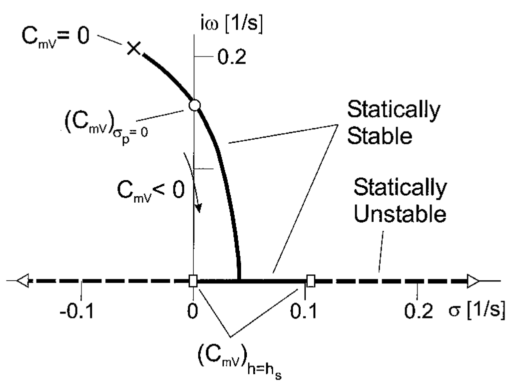

Fig. 1 Thrust offset effects for operation on the backside of the power-required curve (delta wing aircraft).

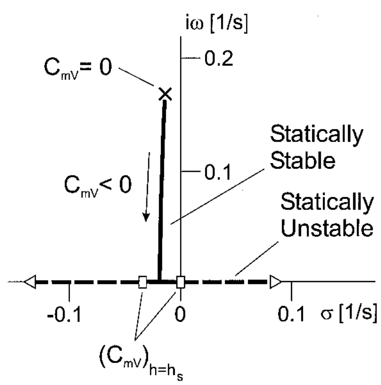

Fig. 2 Thrust offset effects for operation on the frontside of the power-required curve (subsonic transport).

followed by an aperiodic divergence when the negative C_{m_V} values are further increased.

The aperiodic instability caused by thrust offset is of particular concern. This is because it is existent though the aircraft is statically stable. Usually, positive static stability is considered sufficient to prevent aperiodic divergence. The relation between aperiodic instability and positive static stability is also shown in Fig. 1. There is a most unstable aperiodic root for the positive static stability range. It can be approximated by

$$1/T_{\text{aperiodic}} \approx 2(g/V_0)\left[C_{D_\alpha}/C_{L_\alpha} - (1 - n_V/2)(C_D/C_L)\right] \tag{8}$$

and occurs at the static stability limit (zero stability margin $h = h_s$, Ref. 2)

$$\left(C_{m_V}\right)_{h=h_s} = 2C_L C_{m_\alpha}/C_{L_\alpha} \tag{9}$$

By contrast, thrust offset effects at frontside operation introduce neither oscillatory nor aperiodic instability as long as the aircraft has positive static stability (Fig. 2). This implies that the stability boundary value $(C_{m_V})_{\sigma_p=0}$ of Eq. (6) has no meaning for frontside operation.

Flying Qualities Implications

The considered impact of thrust offset effects on flying qualities is twofold. First, it relates to longitudinal speed axis stability, which concerns the relation between static stability and aperiodic divergence. The second point concerns flight-path stability or the power-required curve.

According to current flying qualities requirements such as Refs. 5 and 6, there is a longitudinal speed axis requirement for preventing aperiodic divergence. The requirement is considered satisfied if the aircraft has positive static stability (equivalently, a corresponding control gradient with airspeed). This presupposes a firm connection between static and dynamic stability, implying that there can be no aperiodic instability when the aircraft is statically stable. Though this is almost generally valid, it does not hold for the thrust offset effects as described in the preceding section. As shown in Fig. 1, there is a C_{m_V} range causing aperiodic divergence despite positive static stability. An evaluation of the root locus characteristics of Fig. 1 yields the following approximate relation for the C_{m_V} range in mind ($n_V = 0$):

$$\left(C_{m_V}\right)_{h=h_s} < C_{m_V} < C_L\left(C_{m_\alpha}/C_{L_\alpha}\right)\left[2 - (C_D/C_L)^2\right] \tag{10}$$

As a result, there is an inconsistency of current flying qualities requirements with regard to thrust offset effects yielding aperiodic instability for statically stable aircraft.

The second point concerns flight-path stability. Related flying qualities criteria and requirements address a closed-loop instability problem of the pilot–aircraft system and specify appropriate limits.[5,6] This instability problem concerns the usual piloting technique to adjust flight path with pitch attitude by the use of pitch control only (throttle setting not changed by the pilot). Flight-path stability can be related to the open-loop altitude to pitch control transfer function h/δ_e. Specifically, closed-loop analysis shows that the zero

$$1/T_{h_1} \approx 2(g/V_0)\left[(1 - n_V/2)C_D/C_L - C_{D_\alpha}/C_{L_\alpha}\right] \tag{11}$$

of the h/δ_e transfer function can be used as an indicator of closed-loop stability characteristics. Negative $1/T_{h_1}$ values indicate closed-loop instability. Comparison with Eq. (5) shows that negative $1/T_{h_1}$ values are equivalent with operation on the backside of the power-required curve.

The addressed closed-loop instability exists even if the open-loop system (aircraft alone) is stable. However, the combined effects of thrust offset and backside operation already cause an open-loop instability when

$$C_{m_V} < \left(C_{m_V}\right)_{\sigma_p=0}$$

where $(C_{m_V})_{\sigma_p=0}$ is given by Eq. (6). Such an open-loop instability may contribute to increase the problems associated with flight-path instability.

Summing up the considered impacts on flying qualities, the effects of thrust offset concern two flying qualities metrics. There are degradations with regard to both metrics when the detrimental thrust offset effects are combined with operation on the backside of the power-required curve.

Conclusions

The effects of thrust offset on longitudinal dynamics and related implications of flying qualities are considered. It is shown that significant destabilizing effects of thrust offset are possible for operation on the backside of the power-required curve. In particular, an aperiodic divergence can be introduced in spite of positive static stability. Flying qualities implications are twofold. It is shown that there is an inconsistency as regards a longitudinal speed axis requirement, according to which there is no aperiodic divergence if the aircraft is statically stable. Another implication concerns flight-path stability. Thrust offset effects introducing open-loop instability may contribute to the difficulties of closed-loop flight-path control problems due to operation on the backside of the power-required curve.

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